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Pricing and Hedging with Equity Options and Futures: An Introductory Study

Dr Abhijit R. Chavan
Professor of Chemical Engineering,
Dr Babasaheb Ambedkar Technological University,
Raigad - 402103, M S.
archavan1972@gmail.com

Abstract

This chapter provides a concise yet thorough overview of the pricing and hedging processes essential for equity options and futures, which are key instruments in the broader derivatives market. It describes how these instruments help investors and institutions manage market risk, improve portfolio performance, and exploit arbitrage. Starting with the principles of derivative markets, the chapter elaborates on the economic background of options and futures and draws the structural and contractual differences between the two. The paper then examines pricing theory, with an emphasis on Binomial and Black-Scholes-Merton models for options and the cost-of-carry model for equity futures, to demonstrate how valuation is subject to market expectations, volatility, and interest rates. The applications of hedging are explained using practical strategies such as long and short futures positions, protective puts, covered calls, and collars, with the trade-offs between risk reduction and potential returns highlighted. Practical examples show how portfolio managers employ derivatives to reduce their risk to changes in equity prices and systemic market shocks. The chapter further reflects on the sources of basis risk, the significance of model assumptions, and how liquidity and margin requirements affect hedging effectiveness. It connects theory and practice, providing readers with an understanding of the principles of derivatives pricing and hedging in today's capital markets. It ends with a discussion of new developments in equity-linked products, such as volatility-based products and algorithmic hedging methods, which are shaping the future of risk management and speculative finance.

Keywords: *Presented here are equity derivatives, Option pricing, Futures pricing, Hedging strategies, Binomial model, Black-Scholes model, Cost-of-carry, Put-call parity, Risk management, Portfolio insurance, Basis risk, and derivative markets.*

1. Introduction

It is now a fact that equity options and futures are essential instruments of the new financial markets, as they enable investors, firms, and intermediaries to control exposure to stock price risk in a fine-tuned, flexible manner. The development of the formalisation of option pricing theory in the early 1970s, particularly the seminal paper by Black and Scholes (1973), provided the first model in which option prices could be consistently related to observable market variables in a non-arbitrageable way. Their model demonstrates that the fair value of an option, when subject to some assumptions regarding market completeness, frictionless trading and continuous hedging, can be determined solely based on the dynamics of the underlying stock price, the risk-free interest rate, time to maturity, and volatility, without any input of risk preferences of investors.

India has become one of the most dynamic equity derivatives markets globally over the past 20 years, both in index derivatives and single-stock contracts. The National Stock Exchange (NSE) is ranked among the world's top exchanges in terms of equity index options and futures traded, and index options under the Nifty family are the largest in terms of overall derivatives volumes. Equity options and futures have ceased to be exclusive institutional hedging niche products among Indian investors, but have become directional trading, income-generation, and risk-management tools among high-net-worth and sophisticated retail investors.

The Indian equity derivatives market began to evolve globally after the development of pricing theory, but it utilised it in line with local market conditions, regulations, and microstructures. In 2000, the UAE introduced exchange-traded index futures on the Nifty 50, which were later replicated by the introduction of index options and derivatives on individual stocks, all being settled by strong and vigorous clearing corporations with very strong margin and risk-management systems. Consequently, the Black-Scholes-Merton and binomial pricing models have become a standard part of the risk system of Indian brokers and banks, and day-to-day trading and hedging choices have continued to be influenced by the regulations of the SEBI, lot sizes, contract specifications, and a high proportion of retail and proprietary areas of trading in India.

Merton (1973) generalised and extended this framework by placing option pricing within a more general theory of contingent claims analysis, in which option values are obtained via dynamic replication and no-arbitrage arguments in continuous time. Writing a variety of financial securities and corporate liabilities in terms of portfolios of simple options written on the assets or equity of the firm, in his approach, the treatment of options, corporate debt, and capital structure choices can be made under the same analytical paradigm. According to this view, pricing and option hedging are not restricted to exchange-traded derivatives but extend to the valuation of a broad spectrum of instruments, including convertible bonds and guarantee and insurance-type contracts.

Although the Black-Scholes-Merton equation is written in continuous time and uses stochastic calculus, a discrete-time binomial model developed by Cox, Ross and Rubinstein (1979) is based on the same underlying economic intuition but has a simpler form. In the binomial model, it is assumed that the underlying stock price at any given time will move by either a required increase or decrease in each time period, and an option is valued by building a portfolio of the stock and a risk-free asset that will pay off the option in that state. Again, no-arbitrage is imposed. The benefits of this discrete-time method are especially pedagogical, in that it presents students and practitioners with a way of visualising the mechanics of dynamic hedging and risk-neutral valuation, which can be learned using only algebraic derivations and backward induction, and tends to approach the continuous-time Black-Scholes formula in the limit as the number of steps becomes large.

These three theoretical foundations, the option pricing formula presented by Black and Scholes, the rational option pricing model by Merton, and the binomial model introduced by Cox, Ross, and Rubinstein, are the theoretical foundations of the derivatives practice of modern derivatives. All of them prove that, under no-arbitrage conditions, the value of an option can be determined by the dynamics of the underlying asset, as well as by the feasibility of dynamically trading the underlying asset and a riskless security, and not by subjective investor expectations. This observation suggests a framework in which risk-neutral valuation can be performed, with option values calculated as discounted expected payoffs under a probability measure that incorporates the market price of risk, thereby simplifying both theoretical modelling and numerical computation.

In practice, both the Black-Scholes-Merton and Cox-Ross-Rubinstein pricing models are not purely academic theoretical models; they are closely connected to portfolio rebalancing that traders and risk managers undertake continuously to ensure they achieve the risk profiles they want. As an example, delta hedging to adjust the position in the underlying stock to compensate for small fluctuations in the option's value is the direct motivation for the replication arguments applied in such models to calculate the price of an option. In the same vein, sensitivity measures, or the Greeks, as they are commonly called, that inform day-to-day risk management in the options markets can be seen as by-products of the same analytical frameworks proposed in the original papers by Black, Scholes, Merton, Cox, Ross, and Rubinstein.

This chapter introduces priced and hedged versions using equity options and futures, with a conceptual basis in these classical models and a practical focus on their use. It starts with a description of the simple geometry and economic intuition of equity derivatives. It proceeds to the fundamental principles of option pricing and hedging, as developed in the Black-Scholes-Merton and binomial models. The focus is consistently on developing intuition about the effectiveness of these models, how they are applied to calculate fair values and devise hedging policies, and what the models' limitations mean for those working in markets subject to frictions, discrete trading, and time-varying volatility.

2. Equity Derivatives Fundamentals.

Equity derivatives are financial contracts whose payoff is determined by the value of a particular equity, usually a single company's stock or an index of stocks. Among these, the best instruments of pricing and hedging against equity risk are options and futures. An equity option confers upon its holder the right, but not the requirement, to buy (call option) or sell (put option) the underlying security at a fixed strike price on a prearranged expiration date (European style) or up to a prearranged expiration date (American style), in consideration of an upfront premium. By contrast, an equity futures contract is an agreement between two parties to either sell or buy a security based on an underlying security at a specified price on a specific future date, normally traded on organised exchanges at daily marking-to-market prices and with margin.

These instruments have an economic rationality that is more easily understood with the option pricing theory formulated by Black and Scholes (1973) and refined by Merton (1973). Black and Scholes demonstrated that a European call on a non-dividend-paying stock can be replicated by a continuously adjusted portfolio of the underlying stock and a risk-free bond, suggesting that the no-arbitrage condition uniquely determines the option price, which equals the value of the replicating portfolio. This replication argument shows that an option is not a separate object but a derivative claim, the value of which is closely associated with the dynamics of the underlying equity and the opportunity to transact in the underlying equity lawlessly over time. The Black-Scholes formula, the solution to the corresponding partial differential equation for the dynamics of lognormal stock prices, provides a closed-form expression for the fair value of a European call or put option, given the current stock price, strike price, risk-free rate, time to maturity, and volatility.

Merton (1973) generalised these concepts by placing option pricing within a broader theory of the rational pricing of contingent claims in continuous-time markets, where investors can trade continuously in stocks and bonds and where no-arbitrage holds that the discounted prices of assets are martingales. In this context, a large variety of financial instruments, such as corporate debt, equity, warrants and guarantees, may be regarded as the specific combinations of the simplest options written on the asset or stock of the firm, and it was then discovered that the corporate liabilities themselves can also be decomposed into option-like structures. An example: the equity of a levered firm would be equivalent to a call option on the assets of the firm with a strike price equal to the face value of debt, whereas risky corporate debt would be a risk-free bond less a put option written on the assets of the firm. This contingent-claims interpretation reveals that equity derivatives are both trading instruments and conceptual components of corporate finance and capital structure.

The binomial model, proposed by Cox, Ross and Rubinstein (1979), is a discrete-time version that generalises the key ideas of arbitrage and replication in a simpler mathematical context. In the binomial model, the stock of each little time period may take on two potential values- an up state, a down state and risk-neutral probability distributions are specified to have an expected value of the stock at risk under these distributions equal to the risk-free rate. A value is then computed by working backwards through the known payoff at maturity, calculating the value at each prior node as the discounted expected value under the risk-neutral probabilities, which is also the cost of a locally replicating portfolio of the stock and a risk-free asset. A larger number of time steps and a smaller step size will bring the binomial model to the smooth-time Black-Scholes price, showing that discrete-time replication and continuous-time hedging are both instances of the same no-arbitrage principle.

To have a basic understanding of the framework of equity options and futures, it is also imperative to consider how these contracts are applied in practice for hedging and risk management. Since the payoff of options is nonlinear with respect to the underlying share price, option payoffs can enable an investor to establish payoff profiles that would otherwise be hard to realise solely with stocks and bonds (such as limiting downside risk while retaining upside potential through protective put strategies). Futures, on the other hand, provide a linear exposure to the underlying asset and are commonly employed to reprice the market exposure of a particular equity portfolio at relatively low transaction costs and speed, including index futures hedges to hedge against a general market downturn. Theoretical solutions of Black-Scholes-Merton and Cox-Ross-Rubinstein enable traders to use these solutions (both of which are useful for pricing) and hedging ratios (delta and gamma) as they combine options, futures, and the underlying equities to arrive at these desirable payoff structures.

Pedagogically, starting with the fundamentals of equity derivatives and their applications in the economic sphere prepares the ground for the discussion of pricing and hedging methods in the subsequent parts. This can be better enabled by first elucidating the definition of options and futures, the manner in which they are designed and why they exist, which will help us to more readily encourage the necessity of having rigorous models like those developed by Black and Scholes (1973), Merton (1973) and Cox, Ross, and Rubinstein (1979) and demonstrate how they can be used to translate into practical valuation and risk control tools in the equity markets.

3. Fundamentals of Option Pricing

The calculation of options commences with several fundamental concepts on the value of an option. In the case of an equity option, the key considerations are the current stock price, the strike price, the time to maturity, the stock's volatility, the risk-free interest rate, and any anticipated dividend payments. The value of a call option is higher when the stock price is higher, the time to expiry is longer, or the volatility is higher, since it has a greater probability of finishing in the money. An increase in strike price tends to reduce the value of a call, whereas an increase in risk-free rate tends to raise the value of calls and decrease the value of puts by altering the present value of the strike.

A key distinction is between **intrinsic value** and **time value**. The intrinsic value of a call is $\max(S - K, 0)$, and for a put it is $\max(K - S, 0)$, where S is the current stock price, and K is the strike price. Time value is the extra amount over intrinsic value that investors are willing to pay because there is still time for the price to move favourably before expiry.

Take a real-life Indian example of the Nifty 50 options in March 2026 on the NSE. Assume that the Nifty 50 spot index is trading at 24865 (closing level on

March 23, 2026). A near-month call option with a strike of 24,500.00 is priced at 550, and the risk-free rate is estimated to be the current RBI repo rate of 5.25 per cent per annum. Its expiration period is 15 days (0.041 years).

Current Nifty level S = ₹24,865

Strike price K = ₹24,500

Call premium C = ₹550

Intrinsic value:

"Intrinsic value"=max(S-K,0)=max(24,865-24,500,0)="₹" 365.

Time value:

"Time value"=C-"Intrinsic value"=550-365="₹" 185.

This is an indication that although the call is in-the-money at 365, traders are willing to incur a further 185 to realise a higher gain before expiry, which reflects volatility expectations (India VIX) and the time-to-expiry influence in the Indian market.

Applied as a practice, these fundamentals, that is, determinants of value, intrinsic and time value, as well as no arbitrage relations like the put call parity, are provided to verify that observed options prices are plausible, before resorting to using more sophisticated models, such as the binomial tree or the Black Scholes Merton. They can also be used to understand standard hedging strategies (including protective puts and covered calls) by demonstrating how various options, stocks, and bonds can be combined to form a particular payoff structure.

4. Option Pricing Models

Modern option pricing models offer a systematic means of linking the value of an option to observable market variables such as the price of the underlying stock, its volatility, interest rates, and the time to maturity. The pillars of introductory treatments are dominated by three: the no-arbitrage principle, the replication of the option payoff using the underlying and a risk-free asset, and risk-neutral valuation. Hull introduces these concepts in the textbook by demonstrating that, in an idealised market with frictionless trading, the absence of arbitrage implies that any derivative security must have a price equal to the cost of a portfolio that perfectly matches its payoff. McDonald is no exception and stresses that the point is to build portfolios that pay off in the states of the world in the same way as the option, producing riskless profit when the price fails to equal the cost of replication.

The binomial model is advantageous because it was introduced first, owing to its intuitive form and its use of step-by-step numerical methods. In a single-period world of binomials, the stock price moves up or down by deterministic factors, and the option payoff for each state is easily calculated. Upon finding a solution to a combination of the stock and a risk-free bond that yields these two payoffs, both the hedge ratio and the current option price are obtained. Hull illustrates this argument using multi-period binomial trees, where option values are computed via backward induction, using risk-neutral probabilities in which the expected return on the stock is set equal to the risk-free rate. McDonald devotes multiple chapters to binomial pricing, how it can be used to model American options and path-dependent payments, and to cases where closed-form formulas are not available.

As the number of time steps in the binomial tree increases and the step length decreases, the binomial model approaches the continuous-time Black-Scholes-Merton model. The BSM model has the following assumptions: the underlying stock price is modelled as a geometric Brownian motion with constant volatility, and trading can be conducted continuously without transaction costs or limits. Using the Ito lemma applied to the price of the option as a function of the price of the stock and time. By taking a long position in the option and a short position in the underlying stock, one can obtain a partial differential equation (PDE) that the option price should satisfy. The PDE with suitable boundary conditions can be solved to obtain the well-known closed-form formulas for European calls and puts on non-dividend-paying stocks, which are a primary subject in the chapters on the BSM model in Hull. McDonald presents the BSM formula similarly to his derivation of binomial pricing and reiterates it as an interpretation in terms of risk-neutral probabilities and lognormal stock price distributions.

Practical pricing. In addition to the vanilla European options, the payoffs may be further divided into ends, early-exercise options, and more complex ones, necessitating adjustments to practical pricing. Hull explains how to transform the BSM model to known dividend yields, index options, currency options, and futures options, usually by varying the cost-of-carry term in the pricing equation. McDonald also emphasises that forward and futures prices are directly linked to option value by the parity relationship and the underlying's effective growth rate. In the case of American options on dividend-paying stocks, closed-form solutions are often nonexistent, and binomial trees, finite difference methods, or other numerical schemes are the tools of choice, closing the gap between theory and practical implementation.

Risk management-wise, in addition to being a tool for valuation, option pricing models also yield sensitivity measures used to manage and monitor portfolio risk. The risk management text by Hull discusses the derivation of the four types of Greeks (delta, gamma, vega, and theta) analytically using pricing formulas, and financial institutions subsequently use them to develop hedging schemes and risk limits. McDonald supports this by focusing on delta hedging, which demonstrates the role of model-implied hedge ratios in dynamically rebalancing positions in the underlying to control its exposure to small price changes. Combined with the treatment in Hull and McDonald, this provides students with a consistent understanding of option pricing models as the theoretical basis for both valuation and hedging practice in equity derivatives markets.

5. Pricing of Equity Futures

Pricing of equity futures is normally presented using the cost-of-carry model, which relates the price of futures to the prevailing spot price of the underlying equity and the net cost of holding the asset until the futures maturity. The opportunity cost of capital in an idealised market is that the theoretical price of futures found on a non-dividend-paying stock is the compounding of the spot price with the risk-free interest rate over the duration of the contract. To the extent that the underlying stock or index is dividend paying, the present value of the expected dividends during the life of the contract must be subtracted, or a continuous dividend yield can be added to the cost of carry term. This results in the well-known formula in which the future price equals the product of the spot price and the exponential of the risk-free rate minus the dividend yield, multiplied by the time to maturity. McDonald gives the same reasoning, but in a slightly different form, whereby any discrepancy between the actual futures price and this theoretical futures price opens the prospect of cash-and-carry arbitrage or reverse cash-and-carry arbitrage.

In the case of equity index futures, the cost-of-carry methodology is then generalised to the index, and the index dividend yield is one of the key inputs. Hull observes that portfolio managers use index futures extensively to adjust market exposure and that the relationship between index levels and futures prices is the key to index arbitrage strategies. In practice, traders will compare the observed futures price with the implied fair value from the cost-of-carry model. When the futures price is underpriced, they can purchase futures and sell the underlying index (or a proxy portfolio), and vice versa when the futures price is overpriced. McDonald emphasises that these arbitrage mechanisms keep the spot and futures markets in line. Still, imperfect tracking of the index components, transaction costs, and short-selling restrictions can cause small, time-varying deviations.

The cost-of-carry between futures and spot prices of an Indian equity index and a single-stock future is determined by local interest rates, anticipated dividends and in the case of Indian equity markets and money markets, the particular institutional framework. Under normal market conditions, the prices of Nifty futures usually trade at a small premium to the cash index, due to the positive net of dividend yield and risk-free net of dividend yield in the short term. However, when the market is stressed or large dividend seasons are approaching, the basis may narrow or even turn negative.

Indian futures pricing example:

Consider Nifty 50 index futures expiring in about 1 month (end-March 2026 contract) as of early March 2026.

- Nifty 50 spot index $S_0 \approx$ ₹24,865
- Risk-free rate $r =$ 5.25% per annum (current RBI repo rate)
- Expected annualised dividend yield on Nifty $q \approx 1.2\%$
- Time to maturity $T \approx 28/365 =$ 0.0767 years

The theoretical futures price using the cost-of-carry model is

$$F_0 = S_0 \times e^{(r-q)T}$$

$$\begin{aligned} F_0 &= 24{,}865 \times e^{(0.0525-0.012)\times 0.0767} \\ &\approx 24{,}865 \times e^{0.00311} \\ &\approx 24{,}865 \times 1.00312 \\ &\approx ₹24{,}942 \end{aligned}$$

Thus, the theoretical futures price of Nifty is approximately 24,942. In real-life trading, the near-month Nifty futures may be trading at a small premium or a small discount to this value due to market sentiment, a lack of funds, and transaction costs. This difference between theoretical and actual futures prices is exploited by the cash-and-carry and reverse cash-and-carry strategies in India. However, STT, margin requirements, and execution frictions imply that only sufficiently large deviations can be exploited in practice.

The concept of spotfutures parity, similar to put-call parity in options, is that the combination of a spot position funded at the risk-free rate and a short futures position should have the same payoff as other strategies that impose no arbitrage. Hull systematises these interdependencies, using simple payoff diagrams and algebra, to demonstrate that violating the parity conditions would have enabled arbitrageurs to earn riskless profits. McDonald gives similar parity arguments in his chapters on forwards and futures, which he relates to the larger framework of derivative pricing as a replication-based framework. Such parity is especially significant when valuing derivatives written on futures is required, since it guarantees consistency between futures and the underlying spot market and informs forward curve modelling.

Equity futures risk management is another critical theme in Hull's risk management text. Futures contracts offer a capital-efficient means to hedge general market risk, since only margin is needed, rather than a full cash deposit or other methods, and exposure can be easily adjusted. Hull explains how to calculate the number of contracts in an index futures contract used to hedge a particular equity portfolio by comparing the portfolio value with the futures value, which can be based on a very simple ratio of portfolio value to futures contract value. Hull also describes in his book on risk management how futures-based hedging fits into the overall risk governance framework of financial institutions, how market risk would be measured, limits set, and how hedges would be monitored and rebalanced over time. These applications are reflected in McDonald, where worked examples show how futures are used to temporarily decrease or remove equity exposure, to effect asset allocation changes, or to speculate on future index moves.

Institutionally, aspects such as central counterparties (CCPs) and daily marking-to-market are fundamental characteristics of futures markets that affect pricing and hedging. The Derivatives Text by Hull highlights the role of CCPs in reducing counterparty credit risk and introducing margin requirements, which, in turn, influence the liquidity and reliability of futures prices as hedges. His risk management book also discusses the importance of margin calls, variation margin flows, and possible liquidity strains during high-volatility periods as risks of their own that institutions should address. Another area that McDonald addresses is the mechanism of futures markets, how daily settlement changes the timing of cash flows, and how it may slightly add to the relationship between theoretical and actual prices, particularly when interest rates are stochastic or when margin returns are not equal to the risk-free rate.

In general, the Hull and McDonald approaches to better understanding equity futures pricing provide a unified perspective on the equity futures price, grounded in the same principles of no arbitrage and replication that are applied in the valuation of options. The theoretical and practical explanations of how futures prices are formulated and why they should not be significantly out of fair value over long periods are provided by the cost-of-carry model, spot-futures parity, and index arbitrage. Meanwhile, the introduction of futures into risk management practice, as explained in the risk management volume by Hull, highlights that pricing cannot be divorced from institutional facts such as margining, regulation, and liquidity. The views are critical to anyone seeking a sound background in the pricing and hedging of equity futures, as a subset of broader research on equity derivatives and financial risk management.

6. Hedging Strategies Using Futures

Single-stock futures and equity indices are very popular for hedging because they provide linear exposure to the underlying asset and can be traded at low transaction costs and with low margin. Futures-based hedging is typically introduced in the quantitative finance literature as a control problem: the portfolio manager takes a position in futures to hedge undesirable risk in the underlying cash portfolio, and solves the control problem under model assumptions about correlations and betas. Wilmott explains that dynamical trading in futures may be viewed as a discrete-time approximation to continuous-time hedging, in which the goal is to minimise the variance of hedged returns or to maximise the expected utility of terminal wealth.

The simple equity hedge portfolio uses index futures to hedge systematic risk but does not take significant action to hedge idiosyncratic risk. An example is that when a value V_P is a portfolio with β_P of a future contract that trades a market index with value V_F, the number of desired futures contracts to hedge the market risk is usually given as $N^* = \beta_P \times (V_P / V_F)$, as noted in the introduction to quantitative finance. This formula ensures that any small percentage changes in the index, and therefore in the futures price, are offset by profits or losses on the futures position, so the hedged portfolio is relatively insensitive to overall market movements. Wilmott provides examples of such hedges, where equity portfolios are hedged against anticipated short-term market declines whilst retaining stock-specific exposures.

Greater finesse in futures hedging arises when the hedger's asset is not perfectly matched to a traded futures contract; cross-hedging and basis risk occur. An example of this is a manager who has a portfolio of sectors and uses a broad market index future as a hedge; in that case, the correlation and relative volatility between the sector portfolio and the index must be considered in the hedge ratio. Quantitative therapy demonstrates that the maximum hedge ratio in a minimum-variance sense is proportional to the covariance of asset and futures returns divided by the variance of futures returns, and is also reflected in the literature on applied finance and financial engineering. Carmona has made contributions to the development of options and multi-asset derivatives, noting that imperfect hedges of this type are often observed in practice and that multi-asset modelling is a necessary tool for analysing residual risk when hedging correlated underlyings with a limited futures menu.

Synthetic options and portfolio insurance strategies may also be implemented using dynamic hedging of futures. In portfolio insurance, the manager seeks to ensure that a portfolio has a minimum terminal value and upside participation, similar to a protective put option on a portfolio. A dynamic trade in index futures is one method of estimating this, in which the portfolio value is maintained by buying more equity as the market increases and selling less equity as the market decreases. Wilmott describes these strategies as discrete versions of continuous time option replication, in which futures contracts are modified to replicate the delta of a synthetic put.

The general view of futures-based hedging was given by Carmona in his work on the indifference pricing and hedging, especially in incomplete markets where perfect replication is not possible. In these, the hedger selects futures positions to maximise expected utility, not to remove risk altogether, and utility-based hedge ratios arise, which are inconsistent with classical mean-variance or delta-hedging solutions. This method acknowledges the existence of real-world limitations, transaction costs, and model uncertainty, which render perfect hedging infeasible; therefore, the task is to find an optimal balance between hedging effectiveness and trading costs.

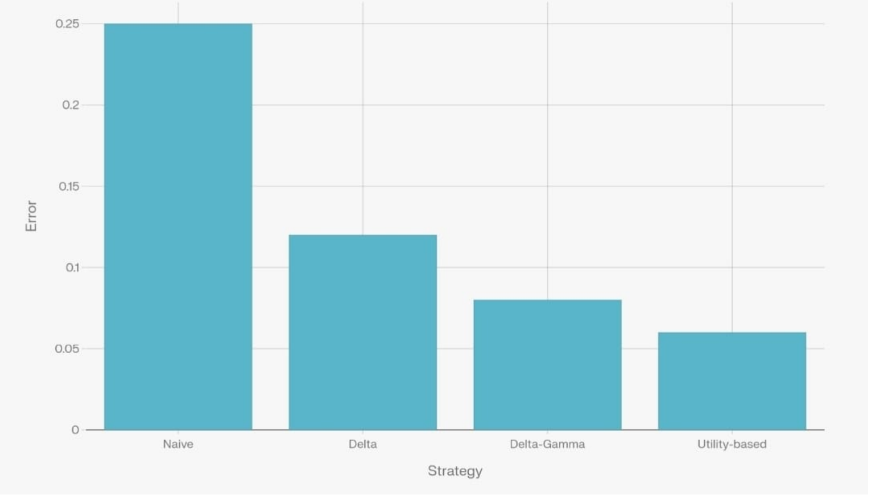
Illustrative table: futures hedge effectiveness

The conceptual differences among hedging approaches can be summarised as follows (values illustrative):

Hedging approach	Objective	Uses correlation structure	Residual risk level	Typical setting
Naive 1:1 futures hedge	Match notional exposure	No	High	Quick ad-hoc index hedges

Hedging approach	Objective	Uses correlation structure	Residual risk level	Typical setting
Beta hedge	Neutralise systematic risk	Yes (beta estimate)	Medium	Equity portfolio vs index
Minimum-variance hedge	Minimise return variance	Yes (covariance matrix)	Low	Cross-hedging, basis risk
Utility-based hedge	Maximise expected investor utility	Yes (full model)	Chosen optimally	Incomplete markets, constraints

This table reflects the progression from simple, rule-of-thumb hedges toward model-based strategies that explicitly incorporate joint dynamics of the underlying and futures, as discussed in Wilmott and Carmona.



The bar chart above shows, in a simplified way, how more advanced strategies (delta-gamma and utility-based hedging) could reduce hedge error compared to naive, simple delta strategies. Although purely illustrative, this is consistent with empirical results in the hedging literature, which indicate that more detailed models with more frequent rebalancing are likely to enhance hedge performance, at the expense of increased complexity and activity.

7. Hedging with Equity Options

The difference between equity option hedging and futures-based hedging is that options have nonlinear payoffs, and investors can develop downside protection and participate in upside in a manner impossible with linear futures. In his quantitative finance textbooks, Wilmott stresses that options and the underlying may be combined to achieve the required payoff profiles, i.e. floors, caps and corridors, which may be converted into particular risk-return trades-offs to the hedger. In his survey of spreads and multi-asset derivatives, Carmona demonstrates that spreads written on asset combinations (spreads, baskets, exchanges) provide even more hedging options in the equity, fixed income, and commodity markets.

The basic approach is the protective put: an investor who holds a security or equity portfolio purchases a put option with a strike price close to its current value, as this will ensure a minimum sale price, whilst still allowing upside should the market increase. This reward is similar to that of portfolio insurance. It may be compared to dynamic futures-based replication: the listed put involves an initial premium but no rebalancing, whereas dynamic replication using futures involves rebalancing and incurs path-dependent costs. According to Wilmott, quantitatively, the protective put is a long position in a risk-free bond plus a call option (by put-call parity). Therefore, the strategy combines insurance and leveraged upside within a single structure.

Another classical options hedge is covered call writing, in which the owner of the underlying stock sells (writes) call options against the position to earn premium income. This approach offers partial downside protection but insufficient upside above the strike price, as the call premium will cover some of the losses if the stock declines, but no protection above the strike price, as the stock would presumably be called at a time when it has significantly increased in value. In many cases, quantitative treatments have shown that covered calls enhance risk-adjusted returns in fairly flat or slightly bullish markets, but not in strongly rising markets, where the opportunity cost of capturing upside becomes very large. The expanded view of options as risk-management instruments by Carmona highlights that alternatives should be evaluated in terms of full distributions of returns, rather than expectations, which is consistent with utility-based or risk-measure-based standards.

Collar strategies involve a protective put and written call, and give the effect of a payoff that is constrained at downside below the put strike and constrained at upside above the call strike. In many cases, collars are designed to be costless or even low cost by selecting the strike so that the call premium approximately cancels the put premium, a construction often discussed in chapters on portfolio insurance. This renders collars attractive to institutional investors who desire set loss limits with no significant upfront cash expenses, and are willing to pay a loss cap to obtain protection. Such structures are described by Wilmott as associated with the engineering mindset: the hedger constructs a target payoff and then solves for the combination of underlying and options that most closely approximates it, subject to market constraints.

More sophisticated hedging arrangements include spread options and multi-asset claims, in which the hedger's exposure depends on the spread or relative performance between two or more equity indexes or stocks. In Carmona and Durrleman's article on the pricing and hedging of spread options, it is shown that such instruments are natural hedging instruments for spread-driven risks (i.e., the underperformance of one equity index relative to the other, or the relative value between sectors). Their analysis also shows that such options typically require dynamic trading across more than one underlying and may involve other vanilla options, as a single spread option is sensitive to the levels of the underlying assets and their correlations. Utility-based and indifference pricing models have been developed in an edited book by Carmona to generalise these concepts to incomplete markets and to determine how a risk-averse investor would optimally hedge equity portfolios when perfect replication is inapplicable.

Lastly, a compromise between pure options hedging and futures-based dynamic strategies is option-based portfolio insurance and synthetic options, which were described in recent chapters on portfolio insurance and contingent immunisation. Within such structures, the manager decides to employ listed facilities, trade futures dynamically, or both to estimate the floor's desire for portfolio value, without control over trading costs or model risk. This selection between the various types of protective puts, collars, dynamic hedging, and synthetic options could therefore be seen to be an optimisation problem over payoff shapes, the cost structure, and the complexity of the implementation, and this would be highly consistent with the financial engineering orientation of Wilmott as well as with the indifference pricing perspective of Carmona.

8. Professional Applications and Case Studies.

The contemporary derivatives practice links option pricing and hedging theory to actual portfolios, trading desks, and institutional risk management. In their work on derivative securities, Jarrow and Turnbull emphasise that pricing and hedging should be developed within a framework that considers all derivative positions, along with a dynamic trading strategy in underlying markets and other instruments. Their discrete-time and continuous-time models demonstrate that the translation of replication and martingale measures into real hedging rules for equity, interest rate, and credit-sensitive claims is direct.

A single central use of discretely rebalanced strategies in equity markets is hedging options and structured products. Boyle and Emanuel examine a discretely adjusted option hedge and measure the hedging error that arises when implementing continuous-time replication with finite trading intervals. They find that the trade-off between transaction costs and hedge accuracy is that higher rebalancing frequency reduces risk and costs, and vice versa. These concepts are directly applicable to equity options desks that hedge large call and put books and exotics by trading underlying stocks, index futures, and liquid vanilla options.

Empirical evidence revealed the existence of strike- and maturity-dependent implied volatilities, which in turn gave rise to the so-called volatility smile in the market following the 1987 crash. The volatility smile and the implied tree by Derman and Kani demonstrate how to generalise the Black-Scholes model to capture market-observed smiles and skews by building an implied local volatility surface that is consistent with option prices. This enables practitioners to price and hedge more complex derivatives (such as barrier options and path-dependent structures) in a manner that reflects the actual market distribution of returns, rather than just the volatility assumption. Implied trees and local volatility models have been extensively applied in equity practice for the structures and management of the risk of structured notes, equity-linked guarantees, and exotic options.

Another abundant use of interest rate and bond derivatives is in this regard. Brennan and Schwartz formulated continuous-time models of bond valuation and the structure of interest rate terms, and they consider the price of bonds as a function of one or more underlying state variables (e.g., short-term rates or ages of major maturities) of interest rates. Their methodology employs stochastic calculus and the no-arbitrage case, as with equity option pricing, but applies it to discounting bonds and the yield curve, which forms the foundation for trading interest rate risk through bond futures, interest rate swaps, and interest rate options. Jarrow and Turnbull further generalise these concepts to credit-risky bonds and credit-sensitive derivatives and offer methods to price and hedge these products when there is default risk on either the underlying or the writer of the derivative itself.

A case study of great relevance at this point of intersection of institutional investing, equity options, and futures is portfolio insurance. In his classic article, "Who Should Buy Portfolio Insurance?" Leland studies the group of investors whose payoff profiles are insured and defines demand in terms of risk behaviour and expectations. He demonstrates that institutions with minimum wealth limits or safety-first goals - as with pension funds and endowments - have a powerful incentive to adopt the strategy of ensuring that portfolio value is kept at a minimum. In practice, this type of insurance can be implemented by buying protective puts, by trading index futures dynamically to replicate option-like payoffs, or by a combination of the two methods in hybrid schemes, as described later in the development of portfolio insurance methods.

These applications show that these theory applications in the previous sections: replication, risk neutral pricing, volatility modelling, and term structure dynamics can be applied to real-world decisions of hedging, structuring, and control of risk in both equity and fixed income markets. A combination of the textbook viewpoint provided by Jarrow and Turnbull, the discrete hedging viewpoint by Boyle and Emanuel, the implied volatility viewpoint by Derman and Kani, the bond models by Brennan and Schwartz, and portfolio insurance provided by Leland, provides a continuum between abstract models and real-world practice.

Table: Illustrative applications and main ideas

Reference	Market focus	Main concept used in practice
Jarrow & Turnbull	Multi-asset derivatives	Unified pricing–hedging framework, martingale methods

Reference	Market focus	Main concept used in practice
Boyle & Emanuel	Equity options	Discrete hedging error, cost–risk trade-off
Derman & Kani	Equity options	Volatility smile, implied local volatility tree
Brennan & Schwartz	Bonds, interest rates	Term-structure modelling, bond pricing, PDEs
Leland	Portfolio insurance	Demand for insured payoffs, dynamic insurance via futures

This table provides an overview of the contributions made by each reference to a particular tool or insight now part of the standard derivatives practice.

9. Conclusion

The literature review above demonstrates that the idea of pricing and hedging based on equity options and futures cannot be isolated from the broader trends in derivatives theory and financial engineering. The work by Jarrow and Turnbull shows that, in both discrete-time and continuous-time models, no-arbitrage, replication, and martingale pricing share a common foundation that provides a rigorous basis for the valuation of a wide variety of derivative instruments. The discussion of discretely adjusted hedges by Boyle and Emanuel leads us to realise that in the real world, hedging can never be precise and that the construction of hedging programs has to trade off the costs of transactions with the risk that remains.

The implied tree structure suggested by Derman and Kani applies classical option-pricing theory to smiley and skewed markets and underscores the importance of tailoring models to market option prices in nonlinear equity risk management. The bond and term-structure modelling by Brennan and Schwartz demonstrates that the same techniques can be applied to fixed-interest markets, where the pricing and hedging of interest-sensitive instruments using bonds, swaps, and derivatives are addressed. The connection between investment goals, market microstructure, and the systemic effects of ubiquitous hedging policies is highlighted by Leland's work on portfolio insurance and by the debate over how to implement this insurance using index futures and options.

In the chapter on Pricing and Hedging with Equity options and Futures, these contributions can be used to fit a consistent story:

- foundational models are used to give fair values and hedging ratios;
- Discrete time and smile consistent extensions are extensions of such models to market realities;
- portfolio-level applications like insurance and risk control are evidence of the way these tools are shaping the institutional behaviour.

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