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Mathematical Pathways to Sustainable Education

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Abstract
A mathematical pathway to cultivate awareness – a systemic need of our time – is proposed. Awareness is taught through educational establishments by mathematical competencies of daily life, quantitatively deployed at micro to macro levels. Grounded in a hierarchy of sustainability and mathematics, three complementary frameworks suggest Education for Sustainable Development infusing Quantitative Literacy across the curriculum. Curriculum Gap – an inherent issue and a mining source of contemporary mathematics. Coupling general national/regional mathematics curricula with quasi-fixed Environmental Science acts as a starting point. Aquatic and Atmospheric projects served with Hydrology and Meteorology learning contexts have yielded fruitful mathematical/quantitative outcomes. Availability of extensive Databases worldwide offers further projects easily coupling Environmental Science and Educational Mathematics for Mathematics with Free resources. A growing awareness of local "Sustainable Development Goals" derives from a macroscopic view of global Sustainability.

Keywords: *Mathematical pathways, sustainable education, quantitative literacy, mathematical modelling, systemic sustainability, interdisciplinary curriculum, data-informed decision making.*

1. Introduction

Education today is not preparing learners for the future challenges. The future readiness idea is a clarion call to rethink what we teach and how we teach it. The aim determines what learners should know and be able to do. Drawing from various sectors, curricular improvements are being pursued that yield citizens who can generate systems, products, and music from diverse social contexts free of multiple error messages (Javier Díez Palomar et al., 2018). In mathematics, a series of linked curricular

pathways is being developed to respond to those dimensions.

Pathways are central to leading learners on rich mathematical journeys. The intended pattern of inter-connections defines the concept. Each destination is the focus of a mathematics community whose goal is to promote inquiry into an ambitious educational goal. Design activities must, therefore, begin with the destination(s). First, future systems are articulated with some rigour. These specify the design activity's mathematical ambitions. Second, the core mathematics needed to enact them is identified. Each specification includes an articulation of entry and exits states, anticipated challenges, and viable trajectories. Pathways are then constructed to enable collectives of diverse learners to make significant progress toward, or alternate approaches to, those destinations. A range of different pathway constellations can then be considered for the specified destination(s).

2. Theoretical Foundations

Sustainability in education refers to the achievement of a society's aspirations for an equitable, stable, and healthy future (Fernando Cruz Quiroga, 2019). These aspirations have been expressed in terms of interdependent social, economic, and ecological considerations, or the 'three spheres' of sustainability. Consequently, educational activities related to sustainability should arise within a framework that recognises and promotes the interconnections of system sustainability. A social-ecological model may provide a useful way of representing the balance to be achieved and the prospective consequences to be expected on a whole-system level for decisions made, activities performed, or policies pursued at the level of any individual sphere. A prerequisite for the acquisition of such high-level decision-making skills includes meaningful opportunities to develop the requisite reasoning tools. Such reasoning is expected to be associated both with quantitative literacy and with mathematical modelling (Dhara & Singh, 2021). Quantitative literacy refers to a person's capacity to form, communicate, and evaluate mathematical conclusions and models grounded in quantitative evidence. Mathematical modelling is the process of translating a real-world situation into and out of mathematics, and it has been recognised as a principal means of developing mathematical literacy. A curriculum featuring quantitative literacy and mathematical modelling is thus a suitable vehicle for supporting education aimed at achieving sustainability. Pedagogical frameworks describing essential topics and accompanying educational practices

associated with quantitative literacy exist—linking sustainability and more specialised forms of mathematical activity through constructing mathematical knowledge, which makes them of particular interest in the context of advancing sustainability for widespread educational implementation.

2.1. Systemic Sustainability in Education

Institutions educate to become part of society. The place of sustainability in students' lives stresses the interdependence between environment, economy and society and prepares them for the unexpected. Facilitators can include sustainability literacy as part of cross-curricular competencies like mathematical literacy instead of offering it as a separate subject (Caetano & Carlos Felgueiras, 2018).

2.2. Mathematical Reasoning as a Tool for Sustainability

Mathematical reasoning qualifies as a fundamental means for addressing various aspects of sustainability. Guidance for selecting focal areas stems from prioritising mathematical practices for societal challenges (Javier Díez Palomar et al., 2018). Central to this perspective, quantitative disciplines encompass a variety of conceptual modelling and data analytic frameworks supporting the description, prediction, exploration, and examination of changeable phenomena.

Instruction centring on mathematical reasoning emphasises the practices, models, and communication capabilities utilised for constructing, critiquing, and refining evidentiary arguments. Adopting a systemic view of sustainability inherently enlarges the scope of mathematical reasoning, for it encompasses a multitude of quantitative practices essential for analysing interconnected networked change, probing viable pathways toward aspirational social goals, and evaluating whether proposed solutions adhere to established requirements or criteria.

2.3. Pedagogical Frameworks for Quantitative Literacy

A pedagogical framework for teaching quantitative literacy forms a necessary complement to the characteristics of mathematical reasoning specified earlier. The selected framework seeks to situate the form of reasoning outlined in the framework within an appropriate pedagogical context, and to relate that framework to both the curricular elements described in subsequent sections and to an overall research design.

Several existing frameworks for teaching quantitative literacy have been

considered. Prince and Simpson (Prince & Simpson, 2016) adopt a constructivist lens that seeks to define educational practices in terms of three fundamental principles: addressing community needs, managing contextual challenges, and promoting engagement. LaTondre (2017) proposes a situative approach with six dimensions: classroom activity, assessments, out-of-school engagement, epistemic framing, social forms, and disciplinary framing. The equations presented in Section 2.2 and accompanying curricular elements fit most naturally within the situative framework, but various mappings indicate that they could also be supported within Prince and Simpson's constructivist framework, as well as within a disciplines-based framework.

For three distinct reasons, however, the situative framework was ultimately chosen. First, mathematical reasoning is a tool that has gained acceptance for being used outside of mathematics, namely in the disciplines of science, engineering, finance, economics, etc. A situative framework recognises the crossing of disciplinary boundaries, while other frameworks are more explicitly framed within a limited selection of disciplines. Secondly, the situative perspective indicates that the interpretation of a quantitative practice may vary according to the context. While they apply the same taxonomic system (see Section 2.2), other possible categories for the practice might improve its descriptive potential. Ultimately, the emphasis on epistemic framing is critical to the importance of allowing learners to make sense of quantitative practices in their own way.

3. Methodological Approaches

A mixed-methods approach, using quantitative and qualitative components, informed by design-based implementation research, is employed to investigate an interdisciplinary curricular intervention linking mathematics and sustainable development. Design processes, pedagogical practices and learning outcomes are documented by data. Quantitative assessment evaluates the alignment to sustainability-related curricular objectives. The student responses were analysed statistically to measure the efficacy of the intervention. (Author), 2019 mentioned in his research design strategies for different communities in different places.

3.1. Quantitative Assessment in Educational Contexts

Quantitative considerations have long been incorporated into educational curricula at all levels. To assess these skills effectively in any education system, three essential requirements must be

fulfilled. The assessment should satisfy the requirements of validity and reliability, and it should also correspond to the learner outcomes of a certain curriculum and pedagogy. Any curriculum supporting quantitative literacy to empower learners with the capacity to contribute to systemic sustainability must have valid and reliable assessment of quantitative literacy tools, processes and documentation that have explicit alignment to quantitatively-oriented sustainability outcomes. The evaluation of curricular interventions started in the region of Ticino, Switzerland, on Early Learning, has been reported (D. et al., 2019) to have developed a new framework for evaluation. Progress reports, which were designed to be rigorous and aligned, were key to assessing the implementation of the framework. The multi-dimensional approach, which is used in Early Learning assessment, could also easily be applied to sustainability curricula.

The scholarly literature describes multiple approaches to modelling that assist learners in developing quantitative reasoning and competencies (Sitabkhan et al., 2018). Several potential paths of curricular design that employ documented modelling approaches align with the aforementioned sustainability objectives to support the design of projects that assist in addressing sustainability issues while also contributing to broader methodological modelling goals.

3.2. Modelling and Simulation in Curriculum Design

From a sustainable education perspective, modelling and simulation activities promote the understanding of dynamic systems since they allow students to create and test their own perceptions of the studied phenomena. Such tasks also develop abilities that are key to sustainability, like reflection, decision-making, and creativity. However, these approaches are still sporadically used in classrooms, mainly due to the wide variety of models available, the complexity of the dynamic systems involved, and teachers' preparedness to implement related activities. Following the Modelling for Kids methodology, a modelling and simulation-based curricular intervention has been proposed to foster awareness of sustainable development in education. It consists of a sequence of georeferenced, multisensory modelling activities designed for the early grades. The provided norms, strategies, and contexts aim to facilitate the integration of modelling and simulation tasks in early education and to enhance the students' comprehension of dynamic systems by rendering the learning process

interactive and engaging through the use of multisensory, georeferenced information (Jorge Brigas, 2019).

3.3. Data-Informed Decision Making

Data-informed decision-making in educational contexts is intended to improve instructional practices, further clarify instructional goals, and identify factors hindering desired learning outcomes (William Foley, 2007). In the current study, input from postsecondary students regarding the importance of curricular elements related to sustainability and mathematics is gathered, alongside analysis of curricular designs promoting these connections across K–12 settings. Input from administrators in diverse locales concerning desired qualities in mathematics curricula is also obtained. Data from learning assessments, both formative and summative, are gathered to ascertain students' acquisition of competencies deemed relevant by participating institutions (M. Ratsavong et al., 2017). Ultimately, the data collected are analysed and translated into concrete recommendations for designers of mathematics curricula at all levels.

4. Curriculum Elements

The learning of mathematics is an essential element in building a world capable of sustaining itself. Mathematics is crucial for comprehending, interpreting, modelling, and addressing society's various problems; hence, it is a vital dimension of every student's education. The idea of sustainability stems from a systemic notion in which mankind's ever-expanding demands on a limited space and resource induce interdependent effects that can devastate human society. Consequently, sustainability education is imperative. Matters of sustainability are highly diversified and context-dependent; examining sustainability issues requires a whole-systems perspective. A few critical mathematical topics come into focus, including statistics, optimisation, mathematical modelling, data visualisation, and uncertainty in data. In teaching these concepts, selected types of interconnected project-based activities have great potential, and an interdisciplinary approach further strengthens the connection between mathematics and sustainability (E. Bickerstaff et al., 2018; Noyes & Sealey, 2011).

4.1. Core Mathematical Concepts for Sustainability

Mathematical reasoning is crucial for sustainable societies. Therefore, core maths concepts and practices need to be defined so that reasoning about complex sustainability issues can take place. The ability to reason statistically and to model complex dynamic systems is

fundamental. Statistics allows for the simplification of excessive information, visualisation of trends and uncertainty, and contrasting alternative explanations of the same event. Most international sustainability data is still collected, reported and communicated at a national level despite globalisation. As a result, it is important to re-focus on the dynamic systems discussed in formal learning, particularly those involving people and their numbers (growing or declining) through drawings and simulations. This will eventually lead to a good grasp of societal issues and interrelatedness. Spatial distribution, pollutants, transport, etc., provide clear opportunities for integrating either science or geography (phenomenon emphasis) and ethics or economics (public policy emphasis). Modelling trade-offs and synergies, or optimisation, is also important during those early years because such trade-offs and markets respond surprisingly universally across vastly different regimes.

†Statistics, optimisation, and modelling are also central to the critical task of data-informed decision making. Environmental, economic, societal, and even more specialised sustainability objectives tend to be pursued simultaneously in even the most elementary planning cases. Optimisation serves as a universal, simple, readily internationalised paradigm for formulating and exploring such questions precisely. Such considerations are also critical for framing questions and interpreting both data and proposed solutions within the enclosed data-space surrounding broader questions. The differential examination of alternative planning strategies offered by the global GIS revolution affords further opportunities for richly quantitative interrogation.

4.2. Applied Projects Linking Mathematics and Sustainability

The following applied projects serve as entry points for mathematically oriented investigations addressing sustainability. Each project fosters a set of skills that transcends disciplinary boundaries, yet may anchor the learning experiences in concrete, socially relevant cases. Each project articulation includes a problem statement, a description of the competencies involved, and a corresponding rubric to guide evaluation (Nicho, 2018).

Analysis of a student's school cycling route.

This project targets a school's particular cycling route. Students evaluate individual sessions of their cycling speed and develop a strategy to optimise overall activity. Through tabulations, graphical representations, and the use of

spreadsheets, they observe that sometimes a longer journey allows them to increase the total activity. They also contemplate the influence of factors such as variation of speed, the number of stops, and safety issues on the planning of future trips.

Monitoring of service leaks.

Students gather observations about the presence of leaks in the taps of their homes. They design a survey to collect similar information from their neighbours and reflect on the percentage of persons suffering these leaks, along with the water volume wasted. They simulate the evolution of both their taps and the ones from their peers. Simulations can lead them to a deeper understanding of the hazards attached to leaks, the differences between household routines, and the value of collaborative efforts.

School waste management.

Pupils inspect the waste produced in their premises, categorise it into suitable classes, and weigh the amounts for a reasonable period. Through such a survey, they discover some aspects of their consumption habits and investigate possible improvements. Using their measurements, they establish a strategic plan to reduce waste on specific materials, compute approximately how effective it has been, and run another survey afterwards. Active students contribute with new experiences they have obtained while increasing the amount of reusable, recyclable, or compostable material.

Climate monitoring through mobile applications.

Students choose freely and install a mobile application aimed at collecting personal information on multiple climate aspects. Many relevant international databases are accessible, and they join global platforms delivering data either automatically or by frequent user surveys. Accessing such information in class helps them understand the complexity of multi-variable climate-research matters, together with the involvement of complementary disciplines like psychology, sociology, or informatics. They deal extensively with issues like censorship on measurements, the credibility of database providers, or even the interpretation of specific climatic variables.

4.3. Interdisciplinary Integrations

Bringing different subjects together ensures a comprehensive insight into sustainability, being all-encompassing and complex. It also ensures mathematics is powerful enough to assess and fix socio-environmental issues, which are not simplistic. According to Liu et al. (2022),

interdisciplinary education and education for sustainable development share several pedagogical principles: disciplinary, discovery and participatory learning approaches and approaches involving group discussions. When an integrated perspective is taken on curricular interventions, it facilitates teaching-learning activities and promotes the development of non-cognitive skills like personal values, ethics, social responsibility, self-discipline, ability to act as change-agents for sustainable development, etc., across disciplines, not just environmental studies. The disciplines that can be made relevant to a sustainability-oriented mathematics education include science, geography, economics, and ethics. The relevant exchange could be intersubjective, implying partnership in learning and doing – through joint projects and collaborative exchanges.

The Ontario secondary school mathematics curriculum outlines relevant competences in such disciplines: using scientific inquiry and design processes to represent, illustrate, and analyze relationships (understanding science), analyzing the effectiveness and fairness of governments and organizations (understanding geography), explaining the scope and impact of economic events and decisions (understanding economics), and assessing the interaction of personal values, society, and the environment (understanding ethical issues). At the postsecondary level, the B. Sc. programs of most universities combine mathematics/statistics with one or more sustainability-related disciplines—in a designated, interdepartmental mathematics curriculum that develops a systemic perspective and models many types of sustainability challenges—enabling diverse collaborations across programs, departments, and campuses.

5. Educational Pathways and Equity

Scalability hinges on the pathway by which students can engage with sustainability-oriented mathematical activity. Some students acquire theoretical grounding in statistical and mathematical models within a traditional mathematics curriculum. This approach affords opportunities for students to engage deeply with a small number of sustainability-related curricular modules while evaluating mathematical development throughout the entire middle-school-to-high-school transition.

Wider accessibility of sustainability-focused mathematical modelling pathways requires attention to prevailing mechanisms of systemic inequity. Students from historically marginalised communities, limited literacy, or low socio-economic status

remain largely uninvited to secondary education mathematics, a prerequisite for access to sustainability-related upper-level courses (Hanselman, 2020). Policies fostering sustainability-oriented curricular reform, professional development, and student modelling experiences must concurrently embody principles of equity, culturally relevant pedagogy, and universal learning design (Lucia Morton, 2017).

Environmental responsibility characterising Collective 2030 objectives, the 2023 United Nations Sustainable Development Goals, and the UNESCO Taipei Declaration—transforming education for the future—resonates with socio-political agendas yet unaddressed within regional educational systems and formal policies.

5.1. Access, Inclusion, and Socioeconomic Factors

Barriers to equal participation in curriculum scenarios are clotted by family socioeconomic status, geographical position, access to technological resources, and organisational context (Javier Díez Palomar et al, 2018). In high-income areas, there is interest and inclusion in mathematics pathways through engaging in sustainability topics. Nevertheless, there are several systemic factors that hinder the accessibility of a complementary design at the lower-income level. A study found that policy documents and pre-service teacher programs for inclusive schooling focus on students with SEN and not on alternative strategies for equity. Similar expectations in mathematics, plus a widespread curricular offer and strong governmental engagement in schools, plus a sufficiently strong offer of departmental resource material, attract participation by students, while additional socio-educational conditions hinder inclusion.

Engagement signals reinforce mathematical significance, extend general competencies, and establish educational transitions to science and engineering content. In disproportioned regions, community-based adapted frameworks introduce sustainability exposure via local firsthand experiences. Underlying course design aligns proposed mathematics elements with recognised theoretical and curricular frameworks—specifically, mathematical modelling, modelling education, and modelling frameworks—and permits co-evolution alongside broader EE programs.

5.2. Culturally Responsive Mathematics Education

Culturally responsive mathematics education focuses on building students'

cultural background. Studies in ethnomathematics have shed light on how gender, socio-economic and cultural groups employ mathematics in their daily life. Teaching is most effective when it draws upon students' experiences. Culture has to play an important role in mathematics education due to differences in access and achievement in students from different groups (Carillon Hyatt, 2013).

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5.3. Global Perspectives and Transferability

Understanding global perspectives helps to design educational pathways that advance sustainability without compromising equity across diverse settings. Although pathways embody specific local characteristics, the principles underlying their implementation are transferable. Whether students pursue general or advanced mathematics, the same elements can enhance education for sustainability. Across diverse geographical and institutional contexts, expanding mathematics and sustainability together support equitable educational access, retention, and achievement.

Mathematics and sustainability curricular models, including the proposed pathways, have common features. The introduction of sustainability topics and practices will occur in three stages. The initial stages will maintain exposure to interconnected concepts. Finally, implementation will take place through discipline-specific studies and practices. System-based experimentation involves making ideas and using models, which are useful not only to predict but also to postulate. Fostering mathematics as a means to resolve local matters, community engagement (Kaerchner-Ober & Dippel, 2011). Sustainability is a big part of maths. Community projects based on data develop the curiosity of students and also help achieve individual and social objectives. Joint projects among stakeholder groupings, including youth and government, help enhance sustainability learning (Sund & Pashby, 2018). Known things are transferred from one area to another in a wings category.

Combining the theory and the practice increases relevance, helps retention and influences choices.

Sustainable education is based on local issues and indigenous initiatives. The transfer of these initiatives without adaptation limits uptake. Many countries effectively improve education for sustainability at lower levels in local contexts at senior secondary and higher levels. The examples can give a glimpse of other regions wanting to improve mathematics through sustainability (Javier Díez Palomar et al., 2018).

6. Policy Implications and Institutional Change

As highlighted in the literature of sustainable-oriented mathematics education, curriculum-oriented initiatives offer the opportunity for a sustainable society issue-relevant support system. The policy and institutional frameworks can be at national, state or regional and institutional or programmatic levels those which provide the base. The three frameworks affect the larger education system in specific ways. The standards and assessments bring focus to students' and educators' priorities and help develop the accountability systems. The teacher development builds better teaching practice and ongoing commitment of teachers to better learning for students. The resource allocation will ensure that the materials, technology, funding and community support are available for the implementation to be scaled up and sustained.

To develop a commitment to sustainability-oriented mathematics education not as a bolt-on but as an ambitious and transformative educational goal, we need to work with policy frameworks that directly shape, steer or influence mathematics education and assurance mechanisms that underpin accountability and evaluation systems (Moore et al., 2018).

Current efforts to establish sustainability and mathematics education standards are already underway in diverse regions. Additional mobilisation surrounding accompanying indicators for mathematics-oriented qualifications will support broader initiatives to enhance mathematics education across formal and informal contexts and build buy-in or leverage existing commitment (Dhara & Singh, 2021).

6.1. Standards, Assessment, and Accountability

Education standards and assessment systems govern the scope and sequence of what is taught and how learning is evaluated. Standards and assessments that specify the rich array of quantitative

practices that youth need to understand and act on sustainability issues are essential to promote a coherent and durable response to the current sustainability crisis. Such practices include modelling with mathematics to self-generate questions, specifying parameters and variables, running scenarios, and interpreting results (M. Christensen, 1998). Questions arise regarding the establishment of policy, professional development, and educator preparation, and how best to implement such systems and support systems.

Systems for assessing the extent to which standards for sustainability-oriented mathematical practices are being achieved—including curriculum materials and use, pedagogical practices, and student learning—are a foundation for providing this broad guidance (De Corte et al., 2012). The opportunity to formulate instructional improvement strategies is informed by collecting and analysing evidence on how extensive and productive these systems have been for other established standards, including the important systemic curricular frameworks, naming both mathematical and sustainability-centred mathematical practices that deserve sustained and equitable attention.

6.2. Professional Development for Educators

To enable the widespread implementation of curriculum reform that improves algebraic reasoning, professional development for practitioners is necessary. The ability to think sustainably is hampered by existing curriculum reforms and underlying gaps in students (Andrew Nivens et al., 2012). Developed in the context of curriculum design, knowledge-capturing instruments provide a framework for systematic exploration of teachers' conceptions of algebraic reasoning and associated pedagogical content knowledge (M. Tyminski et al., 2010). When teachers engage in co-planning, co-teaching, and co-reflecting lessons aimed at developing students' algebraic reasoning over a sustained period, they become more aware of the nature and teaching of algebraic reasoning, their students' thinking, and ways to engage students in algebraic tasks.

6.3. Resource Allocation and Infrastructure

Curricular reform in different education systems will not only require more funding but also timely investment in public education to build data visualisation skills to sustain the initiatives. The establishment of university-community partnerships, including partnerships with businesses and educational institutions, enhances

support for education reforms at local, regional and national levels. Fundamental and extended collaborations, with government, private sectors and universities, which share models, data sets, co-analysis and local-global venues where people can learn from each other (Javier Diez Palomar et al, 2018).

7. Case Studies

Mathematical knowledge is a fundamental human capability for reasoning and communication. Such a capability enables any person, independently of culture or epoch, to formulate models of the world, to establish mathematical relations, and to communicate them readily. There is therefore nothing that forbids any person, under any condition, to develop at least a minimal mathematical capability, to use precisely such a knowledge for the improvement of their own life; to improve or modify mathematics according to personal perspectives; and yet to preserve the favour of mathematics in doing so. Such capability should be guaranteed by a civilization that intends to fulfil its basic ethical duty to offer, promote, devote resources to, and protect the right for each individual, independently of colour, race, nationality, culture, social or economic standing, to live the personal life they freely choose in full accordance with the Temporal Charter of Humanity (Javier Díez Palomar et al., 2018).

The three case studies illustrate different possibilities for mathematics linked to sustainable development goals across educational sectors. The case studies involve different topics, yet they share similar characteristics. Mathematics is used to develop and refine indicators of sustainability; to inform the development of policy frameworks that incorporate and promote sustainability; and to open and sustain community debates about the implications of proposed policies and how those discuss these in times of change.

7.1. Case Study A: Community-Based Sustainability Analytics

A project at Western Springs College (WSC), a secondary school in Auckland, New Zealand, that is developing sustainability literacy and capability in students, teachers and the community, is described in Case Study A (Harré, Henderson, & Morgan, 2021). The project took on a systems framework to generate a common understanding of sustainability. They are trying to set up the school as a system with interdependent sub-systems that relate to education, environment, community, culture, health and economic development. For instance, as a participation-based approach, kura

kaupapa also drew on community concepts of sustainability and equity. This way, they developed cultural-linguistic sustainability. The modelling and sustainability framework became part of a school mission that aims to support high-quality education for the environment, community, culture, and economy. Through strong discussion, inter-disciplinary links in subjects emerged, which benefited teachers in developing a sustainability analysis and learning more about each other's subjects. The students found modelling competencies were improving and that their interest in mathematics was awakening. Other feedback indicated that the collaborative nature of the modelling tools was beneficial to learning across year levels.

7.2. Case Study B: Curriculum Reform in Secondary Education

In 2017, the government of Quebec initiated a curriculum reform aimed at facilitating the development of 21st-century competencies in students by integrating the teaching of sustainability and mathematics in secondary schools. To assess the reform's impact on students' mathematical competencies and its alignment with sustainability-oriented educational outcomes, a longitudinal study was undertaken, focusing on the integration of sustainability issues into mathematics as a powerful means to foster systemic thinking. Baseline data were collected before the implementation of the reform (Noyes et al., 2011).

The reform comprised an education sector-wide integrated sustainability initiative and a more specific intervention within the mathematics domain. To plan these interventions, a model of the mathematical competencies developed in Quebec secondary-school mathematics was considered, as well as the specific competencies associated with the secondary-level mathematics curriculum. Although sustainability is a broadly defined concept, an underlying theoretical framework has been constructed in collaboration with the Quebec Ministry of Education and post-secondary institutions. This framework highlights four dimensions of sustainability: the environment, the economy, society, and culture.

Despite the effort to restrict the definition of sustainability to a specific framework, the intent was complicated by the fact that numerous stakeholder proposals were already on the table. In the particular case of the mathematics curriculum, an even more focused definition was developed to guide the proposed curriculum reform. The additional modelling and simulation focused not only on curriculum design but also on the means to enhance the

planned projects in terms of sustainability and mathematics.

7.3. Case Study C: Higher Education and Sustainable Development Metrics

Higher education institutions (HEIs) are critically positioned to foster sustainable development due to their participatory nature and extensive environmental, social, and economic influence (Dumitrescu et al., 2022). In this context, several frameworks guide the analysis of how they affect sustainable development and the subsequent establishment of strategic objectives for universities, making a considerable contribution towards a global sustainable future (Caetano & Carlos Felgueiras, 2018).

First, the universal plan of action directs guidance to HEIs, as well as it integrates sustainable development principles. In addition, a Support Framework has been developed, which will contain tools to assess the adaptation of their educational offer to sustainable development. According to the evaluation of this framework on 40 Romanian universities, 83% have legislation adopted to support the sustainable development initiative, 80% are searching for national/international partnerships to enhance it, while only 45% have formally allocated responsibility for the sustainable development initiative. These findings demonstrate the significance of considering European sustainable development priorities and indicate that necessary strategic measures may differ between universities.

8. Challenges and Future Directions

The development and evaluation of curricular pathways constitute core exploratory efforts in the presented investigation, where reliability and generalizability of observed findings emerge as critical methodological considerations. In the community-based curricular intervention, for example, simple, self-designed rubrics permit descriptions of numerous local actions toward sustainability, yet findings on students' engagement with mathematical dimensions do not readily extrapolate from the specific nature of socioscientific problem contexts. Likewise, even as noticeable mathematical gains were observable in a significant reform of secondary education curricula, unique urban scenarios and varying institutional forces restrict broader inferences from these experiences. This localised, exploratory scrutiny complements investigations of more systemic, educational-entry network properties and action, yet emphasis on expansive patterns limits attention to individual-scale, collective-community articulation with pupils' family, personal, and community

mathematical practices. Through concentration on the local, the absence of generalised conclusions is matched by fine-grained understandings of activity-characterising properties that can nonetheless motivate attention to parallel, differently located, and broader policy, pedagogical, and curricular considerations.

Data-informed improvement of mathematical pathways embedding community-focused sustainability may confront ethical dimensions spanning pupils' own personal or family lives; community identification and development of joint sustainability and mathematics actions prioritise practices accessible to entire population strata and groups. Guidance from scholars actively engaging in comparable efforts determines both ends and means within ethical boundaries regarding inclusivity, reflection on uneven representation of cultural practices and truths, and equitable support for varied linguistic or nation-state pupils. Instrumental assistance from broader programs amplifies knowledge pathways, community articulations, and insights in these realms while also extending articulation of action systems and their interrelations across diverse geographical, cultural, and linguistic dimensions.

The rapid proliferation of digital devices, artificial intelligence, social media, Web 3.0 environments, and comparable phenomena invigorates endeavours toward more sustainable educational futures and constrains existing efforts to transition job-market-aligned knowledge and skills across urban-regional-civilisation continuities in the uncertain educational present. Fundamental transitions between the (Caetano & Carlos Felgueiras, 2018) relations of systematic pathways, job-market preparation, ontological (i.e., routine-control)-epistemological (i.e., learning-control) knowledge disjunctions, and high-resolution agent-classification macro forms or combinations thus emerge as pivotal fulcra of forward progression. The mathematical model addresses and comments on subsystem and associational aspects about these fundamental transitions and processes already in depth, where spokesperson communities, localised exit-commitment entry-points, multilayer-state-actions, knowledge-inputs, assistance-resources, and community-entry-scale expansions constitute discourse-traceable underlying, action-specific aspects and properties. More publicly accessible features of some systems promote identification and articulation of parallel macro-level pathways, articulate job-market considerations as a motivation for previously unexamined content, and

incentivise active specification or exploration of the mathematically knowledge, technology, or community domains "in a side channel".

8.1. methodological Limitations

Conventional assessments offer limited insight into the implementation of curricular frameworks and innovations on a broader scale, yet evaluation of student performance on sustainability-oriented mathematical pathways occurs at several levels within diverse contexts. Stressing the highly contextual nature of educational initiatives, the subsequent analyses concentrate exclusively on the effects of interventions featuring mathematical pathways for sustainability—elaborated on continually throughout this work. Information is drawn from continuous pedagogical engagement and modelling of curricular artefacts targeting sustainability in community members' professional development—most prominently, school teachers—with extension to other community contexts. Although implications for adherence to a sustainability-oriented educational framework are not pursued in depth, efforts reflect themes common across systemic sustainability, educational policy, and pedagogical frameworks when addressing the implications of modelling, assessment, and data-informed decision-making; each inevitably factors into many forms of mathematics education and corresponding curricular developments.

A framework linking formal mathematical pathways to anticipated pathways for sustainability remains largely unexplored, suggesting an interdependent relationship amidst a multi-grounded interpretation of sustainability, enabling multiple incorporation routes and forms. Since implementation extends beyond merely flexible design provisions, many courses feature coalescences of sustainability-oriented pedagogical approaches, notwithstanding outside deliberation of the matter. Aiding exploration of materials' compatibility, macro-level mathematical modelling broadens the curriculum beyond mathematics alone—moreover, a crucial sustainability device extending into authors' broader engagements (Dhara & Singh, 2021). Influencing vehicle-oriented safety in curricular programmes, addressing security gaps while preserving compatibility with sustainability orientations, akin to broader debates within formal modelling of parameter forms, one of numerous modelling interfaces wherein each modelling priority is independently addressed, yet other attributes and engagements simultaneously remain available.

8.2. Ethical Considerations in Data Use

Data use is often seen as an objective matter of exposing underlying quantitative logic. Likewise, the assumption motivates the prescriptive engineering viewpoint of applied mathematics in which the underlying quantitative logic is explicitly articulated. Yet there is always a question of what data is collected, how it is processed, and for what purpose analytics are intended, and those choices embed value-laden judgements. Immense opportunities are offered by contemporary digital technologies, generating data of ever-increasing volume, variety, and velocity; automating collection, processing, and analysis; and facilitating the improvisation of novel analytics on the fly. Nevertheless, emerging substantive and technical ethical issues arise. A focused discussion of ethical principles and guidelines framed around four core issues—responsibility, privacy, consent, and representation—has recently been proposed (Rycroft-Smith et al., 2022). Efforts to instantiate such ethical concerns articulate potentially damaging consequences deriving from quantitative practices linked to mathematical education in diverse domains, including, inter alios, socio-economic processes, environmental phenomena, governance, and the political dynamics. Efforts to integrate ethical and critical considerations into the teaching of mathematics for sustainability could be reflected within curriculum development.

8.3. Emerging Technologies and Their Educational Implications

Emerging technologies have considerable educational implications. They allow adapting mathematics education to the demands posed by information and communication technology (ICT), promoting student engagement in mathematics through technology-based teaching frameworks, and inducing learning using computer simulations and mobile learning. Such developments alter methods of teaching, ways of assessing students, and the encouragement of conceptual understanding in mathematics (Haapasalo & Samuels, 2017). Technology also enhances environmental awareness as new horizons for education unfold (Drijvers et al., 2017).

Mathematics education faces the challenge of exposing students to mathematically useful problems and environments. The role of technology is crucial in this respect, and information and communication technology holds considerable potential for mathematics education. Nevertheless, challenges remain, including a documented lack of a clear mathematics education vision and a lack of evidence on the direct effect of

technology on learners' mathematical thinking.

9. Conclusion

Mathematics education must play a critical and strategic role in the achievement of sustainable development. Mathematics is needed to provide the analytical skills and systems thinking required to address the many interdependent and systemic challenges today's students will face now and in the future. Coupled with sustainability education, mathematics education provides access to the capabilities needed to manage sustainably critical resources like land, minerals, water, energy, and carbon.

The systematic approach to mathematics education examined in this paper, which investigates the wide range of competencies encompassed in mathematics with reference to sustainability literacy and quantitative reasoning, represents a step forward in theoretical and practical mathematics and sustainability educational design. There are not many systems-based approaches to mathematics education. The frameworks that are discussed represent an important synthesis of recent advances in sustainability and educational theory, mathematics modelling and quantitative literacy. Mathematics educators and curriculum developers, and policy makers involved in developing pathways that enhance mathematics for sustainability, could review and use the proposed frameworks. Common curricular components are already being used in many countries, states, and regions throughout K–12 education. The materials that have been developed for public-sector institutions and second-language private schools show how the curriculum can be developed in varied contexts while conforming to the proposed components.

Education systems worldwide are committed to the Sustainable Development Goals (SDGs). The proposed mathematics educational materials systematically link mathematics to the SDGs and represent an important step towards fulfilling the promise of Mathematics for Sustainable Development.

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